# Technical Notes

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## High-Intensity Point Perturbation in a Two-Temperature MHD Plasma

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### Introduction

THE development of self-generated nonuniformities has been the focus of many studies in MHD power generators, but relatively few studies have dealt with externally induced nonuniformities.[1] This paper deals with the development of a nonuniformity initiated by a very localized production of a highly ionized spark in an otherwise uniform MHD plasma. The MHD device is an infinitely segmented linear generator operating in the two-temperature mode at a high value of the Hall coefficient. It is found that this very localized perturbation can rapidly develop into a nonuniformity that distorts the distribution of the current to form a high-density current filament. This growth of nonuniformity is found to affect the current distribution over a distance comparable to the distance between electrode walls.

The present study was prompted by an experimental study described by Bosma et al.[2] In this experiment, a high-intensity Q-switched laser beam with a power density of $10^5$ MW/cm$^2$ was directed along the lines of the magnetic field in a quiescent plasma region between two current streamers. This high-density beam created a spark with a high-level ionization which was assumed to drift with the plasma velocity. Under this assumption, the measurement of the spark drift rate would therefore yield a measurement of the gas velocity. However, the measurements indicated that the spark drift rate was 5% lower than the velocity of the streamers, which move at a velocity close to the average plasma velocity. Therefore, the motivation of this paper was not only to ascertain the effect of an initial point perturbation on the plasma behavior but also to help explain this apparent disparity.

### Physical Model

The nonuniformity associated with the width of the laser beam is very small compared to the characteristic dimensions of the channel. The beam is aimed along the line of magnetic fields in the axial plane of the channel. Since the ionization level is constant along a line of magnetic field, the problem can be treated as two-dimensional. The unperturbed MHD plasma flows in an infinitely segmented linear generator operating in the two-temperature mode with a constant resistive loading.

The existence of the initial high-conductivity spot results in several effects: 1) a concentration of Faraday current through the high-conductivity spot, 2) the high-conductivity spot shorting the local Hall field, resulting in the formation of two eddy current cells with counter-rotating currents, and 3) the superimposition of the Faraday current with the Hall driven eddy currents resulting in the formation of a current streamer in an elongated spot tilted relative to the unperturbed Faraday current. Just outside the spot, the variation of the local Faraday current density results in a current concentration with a larger tilt angle.

The evolution of the current and the hot plasma after the initial spark is a highly nonlinear time-dependent problem that can be modeled through a system of equations including Ohm's law, the unsteady electron energy equation, the kinetic equations for the production of seed and argon ions, and the equations describing the conservation of charges as herein described.

### Mathematical Model

#### Steady-State Homogeneous Plasma

Prior to the formation of spark, a steady-state homogeneous Cs-Ar plasma can be determined by assuming Saha equilibrium once the operating conditions and geometrical shape of the MHD channel are known. The main governing equations are Saha equations for seed and working gas, neutrality of charges, electron energy equation, and Ohm's law expressed as

$$I(Re+Ri) = -uBh \tag{1}$$

where $h$ is the channel height and $Re$ and $Ri$ are external load and internal plasma impedance, respectively.

The hot spot, located at the center of the segment chosen for calculation, is imposed on the uniform plasma. The electron temperature in the spark is 11,000 K, which corresponds to a very high electron density compared to the electron density in the surrounding uniform plasma.

#### Time-Dependent Nonequilibrium, Inhomogeneous Plasma

As soon as the spark has been formed, a nonequilibrium model developed by Hara[3] is used, except that it is modified to handle the large spatial gradients of electron density and electron temperature. Therefore, the equation for the current stream function $\psi$ becomes

$$\nabla^2\psi + A\frac{\partial\psi}{\partial x} + B\frac{\partial\psi}{\partial y} = \frac{k\sigma}{e}\left(\frac{\partial T_e}{\partial y}\frac{\partial \ln n_e}{\partial x} - \frac{\partial T_e}{\partial x}\frac{\partial \ln n_e}{\partial y}\right) \tag{2}$$

where

$$A = \sigma\left[\frac{\partial}{\partial x}\left(\frac{1}{\sigma}\right) - \frac{\partial}{\partial y}\left(\frac{\omega\tau}{\sigma}\right)\right]$$

$$B = \sigma\left[\frac{\partial}{\partial x}\left(\frac{\omega\tau}{\sigma}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\sigma}\right)\right]$$

and the right-hand side of Eq. (2) includes the contribution of the electron pressure gradient.

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The electron temperature is obtained from the electron energy equation neglecting the radiative loss:

$$\frac{J_x^2+J_y^2}{\sigma}=\frac{3}{2}\delta m_e n_e k(T_e-T_g)\sum_j \frac{\nu_{e-j}}{m_j}$$
$$+\left(\frac{3}{2}kT_e+\epsilon_{ia}\right)\frac{dn_a^+}{dt}$$
$$+\left(\frac{3}{2}kT_e+\epsilon_{is}\right)\frac{dn_s^+}{dt} \qquad j=a,a^+,s,s^+ \tag{3}$$

and the ionization levels of both the working fluid and the seed are determined as functions of time from the solution of a system of equations formed by the conservation of charges and the kinetic rate equations for each species. The length of each time step $\Delta t$ is the smallest value obtained from the following expression at all grid points:

$$\Delta t=\mathrm{MIN}\left\{Cr\frac{\Delta x}{u+c},\quad B\left[\frac{\partial}{\partial n_i}\left(\frac{dn_i}{dt}\right)\right]^{-1}\right\}$$
$$i=a^+,\ s^+,\ e \tag{4}$$

where $Cr$ is the Courant number and is less than unity, $c$ is the sonic velocity, and $B$ is a constant close to unity.

## Results

The operating and geometrical conditions adopted in the calculations are close to the experimental conditions in Ref. 2 as listed in Table 1. Also listed are the initial equilibrium properties of the hot spark and the properties found after 0.33 $\mu$s.

The development of current streamlines is shown in Figs. 1a–1e. The effects resulting from the existence of a high-conductivity spot described in the physical model can easily be seen. The Faraday current density in the hot spot region after 0.33 $\mu$s is found to be about 15 times larger than it was in the initial uniform plasma. The Hall current density inside the spot is 2.5 times the unperturbed current density. It can be seen that as a function of time, the spark becomes elongated and tilted relative to the Faraday direction. In a series of photographs of the evolution of the spark taken at successive times in Ref. 2 and reported in Ref. 4, the spot was found to evolve from a circular shape to an elongated ellipse, with its main axis tilted relative to the Faraday direction. This is consistent with the present results. The high-intensity Faraday current in the spot shows that the Lorentz force acting on that volume of plasma is larger than the body force acting on the bulk of surrounding gas. Therefore, the externally induced spark should decelerate faster than the surrounding plasma.

The existence of Hall-driven eddy currents increases the dissipation in a wide region and increases the plasma internal impedance, resulting in a significant reduction of the generator output. The impedance increases with time as the eddy currents develop. The distribution of current density (Fig. 2) entering the uniform loads indicates a reduction of the current of 18% in the center region and of 3% away from the spot after 0.33 $\mu$s.

**Table 1 Operating, geometrical, and initial equilibrium conditions for calculations**

| Operating conditions for Ar-Cs MHD plasma | |
|---|---|
| Stagnation temperature | 2000 K |
| Stagnation pressure | 9 bar |
| Magnetic field | 5 T |
| Seed fraction | $1\times10^{-4}$ |
| External load | 5 Ω |
| Mach number | 1.9 |
| Gas velocity | 1065 m/s |

| Geometrical conditions for MHD channel | |
|---|---|
| Length in flow direction | 0.02 m; divided into 29 segments |
| Channel height | 0.10 m; divided into 19 segments |
| Electrode area | $4\times10^{-3}$ m$^2$ |

| Properties of steady-state uniform plasma before perturbation | |
|---|---|
| Electron temperature | 2552 K |
| Trans. current density | $2.33\times10^4$ A/m$^2$ |
| Electron density | $1.42\times10^{20}$ m$^{-3}$ |
| Elec. conductivity | 34.5 mho/m |
| Hall parameter | 7.59 |

| Properties in the hot spot | Initial | $t=0.33$ $\mu s$ |
|---|---|---|
| Electron temperature | 11,000 | 9221 K |
| Trans. current density | $2.66\times10^4$ | $3.40\times10^5$ A/m$^2$ |
| Electron density | $2.57\times10^{23}$ | $4.88\times10^{22}$ m$^{-3}$ |
| Elec. conductivity | 2000 | 925 mho/m |
| Hall parameter | 0.24 | 0.59 |

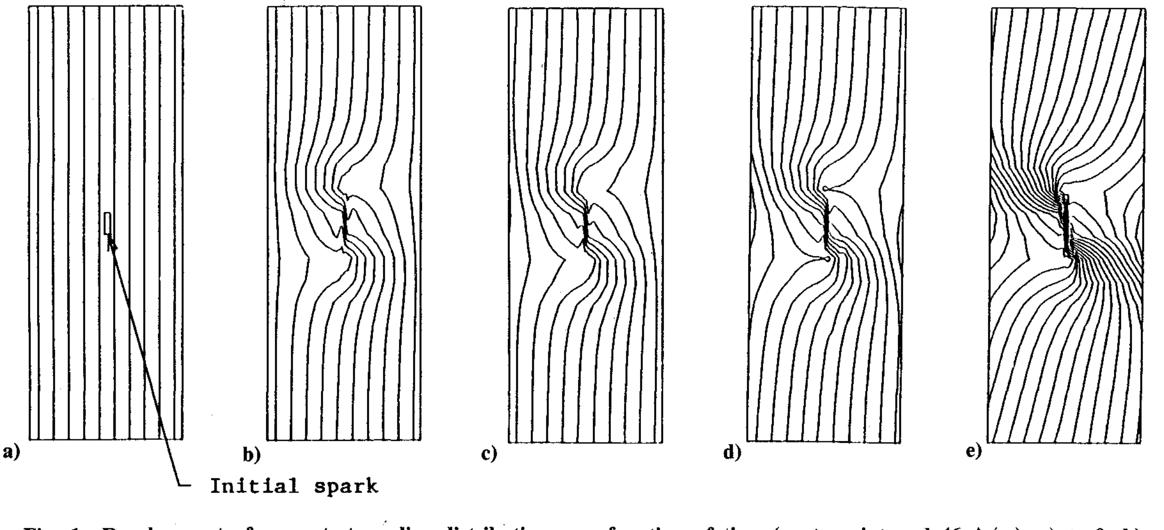


**Fig. 1 Development of current streamline distribution as a function of time (contour interval 46 A/m): a) $t=0$, b) $t=5.25\times10^{-8}$, c) $t=1.09\times10^{-7}$, d) $t=1.93\times10^{-7}$, e) $t=3.31\times10^{-7}$ s.**

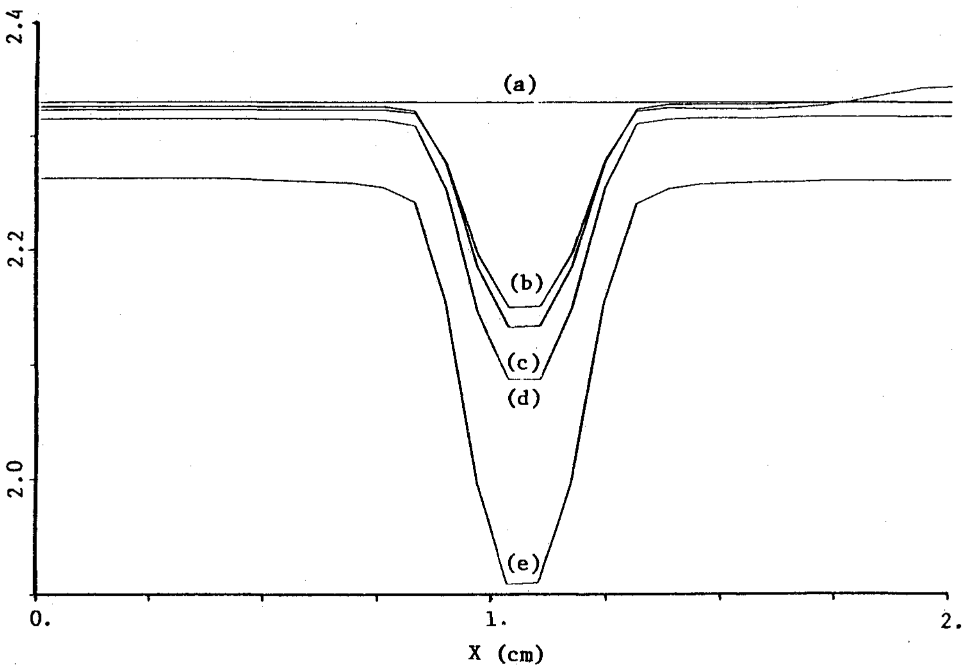


**Fig. 2 Distribution of the generator current output: a) $t=0$, b) $t=5.25\times10^{-8}$, c) $t=1.09\times10^{-7}$, d) $t=1.93\times10^{-7}$, e) $t=3.31\times10^{-7}$ s.**

## Conclusions

1) The nonuniformity induced by an externally formed hot spot is found to deeply affect the current distribution over a distance comparable to the distance between electrode walls. Therefore, the existence of this nonuniformity can significantly influence the generator output.

2) The high current concentration in the hot spot region shows that the spark grows into a strong gas discharge with a current density of 15 times the unperturbed current density. The resultant high intensity of the Faraday current in the spot indicates that the Lorentz force acting on that volume of plasma is larger than the force acting on the rest of the plasma. Therefore, a hot spark created externally can be decelerated more than the surrounding working fluid. This provides a likely explanation as to why the spark velocity was found to be less than the streamer velocity in the experiments of Ref. 2.

## Acknowledgment

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## References

[1]Louis, J. F., "Effect and Nature of Nonuniformities in Nonequilibrium MHD Generators," 9th International Conference on MHD Electrical Power Generation, Japan, Nov. 17-21, 1986.

[2]Bosma, J.N.C., Veefkind, A., Uhlenbosh, J. F., and Rietjens, L.H.Th., "Experimental Investigation of the Gas Dynamic Interaction Between Streamers and Background Gas in a Noble Gas MHD Generator," *Proceedings of the 23rd Symposium on Engineering Aspects of MHD*, 1985.

[3]Hara, T., Veefkind, A., and Rietjens, L.H.Th., "Numerical Simulation of the Inhomogeneous Discharge Structure in Noble Gas MHD Generators," *AIAA Journal*, Vol. 20, Nov. 1982, pp. 1473-1480.

[4]Veefkind, A., Poster Presentation at the 9th International Conference on MHD Electrical Power Generation, Japan, Nov. 17-21, 1986.

## ERRATA

• "Swirling Nozzle Flows," Vol. 3, No. 4, 1987, pp. 342-349. Figures 4 and 6 were transposed.